Operational Amplifiers



Op amps have many applications. They are used in a wide variety of mathematical applications as a fundamental element of a mathematical circuit. Some more complicated models of an op amp have different properties such as input offset voltage, dc gain, bias current, input and output resistance. To account for these properties, more complicated models are required. This section will ignore practical and more advanced models and focus on the ideal operational amplifier.

This module introduces the operational amplifier in a circuit. We will explore the purpose of an op amp, look at the characteristics and examine examples of it. This section will focus on the ideal op amp and show circuits with op amps designed to fulfill a purpose.

Ideal Operational Amplifiers

Ideal op amps consist of 5 main connections: Two connected voltages which act as the positive and negative voltage supply V_{S-} and V_{S+} (often these are not shown in circuit diagrams), two other inputs act as the *inverting* and *non-inverting* inputs V_{-} and V_{+} , and an output signal V_{out}



Figure 1: Ideal op amp

We use the ideal op amp in models to replicate the reaction of a real op amp in the circuit. The ideal operational amplifier exhibits linear behaviour and must abide by three conditions to be considered ideal.

$$|v_0| \le v_{sat} \qquad |i_0| \le i_{sat} \qquad \left|\frac{d}{dt}v_0(t)\right| \le SR$$
... Eq. (1.1-1.3)

Where SR is the *slew rate limit*. This is the fastest possible exchange of electrons that the op amp can endure, usually in volts/second. Similarly with v_{sat} and i_{sat} , this is the maximum voltage and current that the op amp can function with, also reffered to as *saturation*.

In an ideal op amp, the current entering the inverting and non-inverting nodes are zero. In addition, the node voltages at the input nodes are equal.

$$i_1 = 0$$
 $i_2 = 0$ $V_+ = V_-$... Eq. (2.1-2.3)

The output voltage of an op amp is governed by the following equation.

$$V_{out} = A_{OL} \cdot (V_+ - V_-)$$

... Eq. (3)

The difference between the non-inverting and inverting inputs, V_{+} and V_{-} respectively, are

multiplied by an **open-loop** gain, A_{OL} . This gain is usually a very large number so any small changes in the voltage affect the output. This contributes to the unpredictable nature of an op amp.

Open and Closed Loop

An operational amplifier is often connected to a circuit with the output connecting to an input in a way that causes feedback. This is called a closed loop and it is often the way op amps are implemented. This ensures predictable operation and also greatly reduces the effect of the gain. When negative feedback is used, the feedback network rather than the op amp becomes the determining factor for the output and gain.



Figure 2: Closed loop, negative feedback op amp

Here, the effects of the resistors will play a large impact on the behaviour of the op amp. It will establish an equilibrium as the output voltage V_{out} forces the inverting voltage to be equal to the non-inverting V_{in} voltage, due to the feedback. The output will always try to force the input voltage to be equal to each other.

An open loop system does not have a feedback network. The inputs do not depend on the output voltage and the op amp works simply as a comparator. The gain that the output experiences is known as the open-loop gain, A_{OL} .

Nodal Analysis with Operational Amplifiers

It's much more convenient to use nodal analysis on circuits that contain ideal op amps. Three things to remember while undergoing analysis include,

- 1. The node voltages at the input of the amplifier are equal. We can eliminate one of these voltages in the KCL equations.
- 2. The currents leading into the ideal op amp are both zero.
- 3. The current out of the op amp is not zero. Applying KCL at the output node is not necessary because it adds another unknown to the equation. Unless this is asked for, it is not required.

Otherwise, apply KCL normally and manipulate the equations to recieve the desired results.

Demonstration of Op Amp Analysis

This circuit is modelled with an ideal op amp. Determine the output voltage v_0 in terms of the input voltages.



Figure 3: A difference amplifier circuit

Examine the non-inverting node and apply KCL.

$$\frac{v_2}{30\,000} + \frac{v_2 - v_b}{10\,000} + i_2 = 0$$

We can eliminate i_2 , substitute in $v_2 = v_1$ and solve the equation for v_1 .

 $v_1 = 0.75 \cdot v_b$

At the inverting node, the KCL equations are,

$$\frac{v_1 - v_a}{10\,000} + \frac{v_1 - v_0}{30\,000} + i_1 = 0$$

Eliminate i_1 and substitute in $v_1 = 0.75 \cdot v_b$.

$$\frac{0.75 \cdot v_b - v_a}{10\ 000} + \frac{0.75 \cdot v_b - v_0}{30\ 000} = 0$$

Solving for v_0 reveals

 $v_0 = 3 \cdot (v_b - v_a)$

This is called a difference amplifier because the output is dependent on the difference between the input voltages.

Op Amp Applications

Operational amplifiers were originally used for mathematical operations, as the name suggests. However, there are many other ways that an op amp can be integrated into a circuit.

A voltage follower, or buffer amplifier, will eliminate the current between two elements and copy the voltage. This is useful when two circuits are connected and it is undesirable to have the second circuit influence the first.



Figure 4: A voltage follower connecting two circuits.

This circuit demonstrates the function of a voltage follower. Circuit 1 is unaffected by circuit 2. Removing the op amp and circuit 2 and connecting a voltmeter across circuit 1

would reveal the same potential as v_1 in figure 4 has.

Other op amps have been implemented in circuits for various purposes. A current to voltage converter is a circuit that uses an op amp to directly convert an input current into a voltage. A summing amplifier can add input voltages and force out an equivalent voltage. Negative resistance converters cancel out a load resistance to provide the ability for an op amp to output a high output current. There are many uses for op amps that take advantage of its behaviour in a circuit.

Controlling Amplification

Consider the following circuit diagram containing an op amp.





This circuit has a voltage source v_{in} that provides power to the circuit. An operational amplifier circuit is connected to this source, and a load R_L is applied. How does the op amp circuit affect the voltage supplied to the load?

Constants

restart;

 $v_{in} := 12$: Volts $R_L := 100$: Ohms $R_1 := 1$: Ohms $R_2 := 5$: Ohms

An op amp will amplify the voltage by some constant, *K* such that $v_{out} = K \cdot v_{in}$. Different implementations will affect the circuit differently. There are 4 unique cases to consider: K < 0, 0 < K < 1, K = 1 and K > 1. We do not need to examine K = 0 because this can be achieved without an op amp with an open circuit.

Alter the desired amplification of the op amp and observe how it affects the voltage across the load. The magnitude of the amplification can be changed by editing the value of R_1 and R_2 as well. The circuit and its name are shown to visualize the required circuit.



Table 1: Op Amp Design



References:

R. Dorf, J. Svoboda. "Introduction to Electric Circuits", 8th Edition. RRD Jefferson City, 2010, John Wiley and Sons, Inc.